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A. Klatchko, B.C. Choudhary and T. Huehn

Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

University of California
Riverside, California 92521

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A. Klatchko, B. C. Choudhary and T. Huehn
University of California Riverside CA 92521
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Abstract

Following the stochastic nature of the multiple Coulomb scattering process, we derive a formal solution for the parameterization of the errors that are inflicted on the particle trajectory. Our approach confirms an earlier but non general solution [1]. Based on our procedure we construct a random walk algorithm to estimate these errors for any number of radiation lengths, $L_\circ$. Using this algorithm to study the error estimate as a function of $L_\circ$, we find good agreement with the Highland formula [2].

Introduction

Multiple Coulomb scattering (MS) introduces small deviations into the track parameters compared with those of an unscattered track (i.e a particle traversing the vacuum). The effect is usually described by an angle, Θ^{MS} [3] and a corresponding displacement in the position, ϵ [4]. It is usually assumed that the error on the physical process of measurement (the resolution) and the MS errors are independent. Also note that the MS process can be decoupled from energy losses.

As a stochastic process, the probability for a scattering event, denoted by the state $X(t)$ in the phase space, to take place at time $t_\circ$, depends only on the physical condition in the immediate past, at time $t < t_\circ$. This is formally described as a convolution of local probability density functions satisfying the Chapman - Kolmogorov identity,

$$\rho_2(X,t|Y,s) = \int \rho_2(X,t|\xi,u)\rho_2(\xi,u|Y,s)d\xi,$$

where $\rho_2(X,t|Y,s)dX$ is the probability that the event $X < X(t) \leq X + dX$ occurs at time t, given that $X(s) = Y$ for $t > s$. The subscript, '2', emphasizes the fact that only the state in the immediate past matters. Traversing a material of thickness X, the particle undergoes successive small-angle deflections *symmetrically* distributed about the incident direction. Applying the central limit theorem of statistics to a large number of independent scattering events, the distribution of the scattering angle can be approximated by a Gaussian. The mean squared MS angle is defined as, $\langle\Theta^2\rangle = n\langle\theta^2\rangle$, where n is the number of collisions (n is proportional to the number of atoms in the material) and $\langle\theta^2\rangle$ is the mean squared angle of a single scattering event defined as [5]:

$$\langle\theta^2\rangle = \frac{\int \theta^2 \frac{d\sigma}{d\Omega} d\Omega}{\int \frac{d\sigma}{d\Omega} d\Omega}$$

with $\frac{d\sigma}{d\Omega}$ the differential (Rutherford) cross section for a single scattering event. The accumulative scattering angle is estimated by the *rms* of this Gaussian distribution, $\Theta^{MS} = \sqrt{\langle\Theta^2\rangle}$.

This paper deals with the estimation of the errors of the track parameters due to MS, in the milieu of track reconstruction in high energy physics detectors. At a fixed plane of measurement, a track is locally defined by five parameters; two position coordinates, two direction cosines (or angles), and the radius of curvature (when there is a magnetic field), which is proportional to the momentum. In Cartesian coordinates, one has a five dimensional vector, $\bar{V} = (x, \hat{x}, y, \hat{y}, \frac{1}{p})$. Note however, that the parameterization of the track and the errors of its parameters in one coordinate system can always be transformed to another coordinate system appropriate to the detector geometry. It is thus sufficient to evaluate the errors for one set of parameters, for example in the Cartesian system.

In order to estimate the MS effect on the track parameters, it is necessary to evaluate the errors in a given plane of measurement - location b, after the particle has traversed a material with a given thickness, X, corresponding to a number of, $L_\circ = \frac{X}{X_\circ}$, radiation lengths, that is located between the b and a planes of measurement.

We organize the paper in three sections. Based on a statistical approach, the first section describes a formal solution of the problem of the MS error parameterization for a material with any number of radiation lengths. In the second section we use our solution to construct an algorithm for a Monte Carlo study of the MS errors for various $L_\circ$. Finally in the third section, we summarize our results.

A formal solution for the parameterization of the MS error

Let us break the trajectory of the particle traversing the material in the detector into a series of quasi straight lines, $aa', a'a'', \dots$, each with an infinite radius of curvature, such that the trajectory that associates the two locations a and b can be described by a sum of these straight lines. To first order the scattered direction cosines at location b are described by:

$$\hat{\mathbf{x}}^b = \hat{\mathbf{x}}^a + \delta\hat{\mathbf{x}}^{ab} \tag{1}$$

with, $\delta\hat{\mathbf{x}}^{ab} = \sum \delta\hat{\mathbf{x}}^{a'a''}$, a sum over a series of small random direction vectors. Our aim is to estimate the quantities $\delta\hat{\mathbf{x}}^{ab}$. The solution to this problem described in [1] is not unique, we therefore wish to derive the most general solution in the spirit of the stochastic nature of the MS process.

On the single scattering event level we may assume that the process is elastic. Energy loss corrections for a finite path length, can be imposed independently by reducing the particle momentum as a function of the length of the traversed material. Since each scattering event is essentially a rotation, we apply a series of successive infinitesimal rotations to the incident direction vector, $\hat{\mathbf{x}}^a$. In general, to rotate a vector in space, one needs 3 Euler angles but we may neglect the translation parameters. After each scattering event, i, that occurs between the planes a and b, the direction vector $\hat{\mathbf{x}}^i$ is rotated by the infinitesimal rotation matrix:

$$\epsilon^i = \begin{pmatrix} 0 & \omega_3^i & -\omega_2^i \\ -\omega_3^i & 0 & \omega_1^i \\ \omega_2^i & -\omega_1^i & 0 \end{pmatrix} \tag{2}$$

where, ω_α^i are small stochastic variables. The scattered direction vector after the i^{th} scattering event is thus:

$$\hat{\mathbf{x}}^{i'} = (\mathbf{I} + \epsilon^i)\ \hat{\mathbf{x}}^i \tag{3}$$

with $\mathbf{I}$, the 3×3 unit matrix. After n scattering events the initial direction vector, $\hat{\mathbf{x}}^a$, has gone through n independent rotations such that the direction vector in location b is described by:

$$\hat{\mathbf{x}}^b = \prod_{i=1}^{n} [\mathbf{I} + \epsilon^i]\ \hat{\mathbf{x}}^a \tag{4}$$

Note that second (and higher) order terms like,

$$\sum_{j=1, k \neq j}^{n} \epsilon_{\alpha\beta}^j \epsilon_{\beta\gamma}^k, \quad etc.$$

average to 0 fast enough, and can thus be neglected. This leads to:

$$\hat{\mathbf{x}}^b = [\mathbf{I} + \sum_{i=1}^{n} \epsilon^i]\ \hat{\mathbf{x}}^a \tag{5}$$

Let us define the resulting rotation matrix after n Coulomb scattering events as:

$$\overline{\epsilon} = \sum_{i=1}^{n} \epsilon^i \tag{6}$$

The effective rotation of the direction vector after the MS process is therefore:

$$\hat{\mathbf{x}}^b = [\mathbf{I} + \overline{\epsilon}]\ \hat{\mathbf{x}}^a \tag{7}$$

with the resulting matrix, $\overline{\epsilon}$, given by:

$$\overline{\epsilon} = \begin{pmatrix} 0 & \overline{\Omega}_3 & -\overline{\Omega}_2 \\ -\overline{\Omega}_3 & 0 & \overline{\Omega}_1 \\ \overline{\Omega}_2 & -\overline{\Omega}_1 & 0 \end{pmatrix} \tag{8}$$

where the effective rotation angles, $\overline{\Omega}_\alpha$, are Gaussian distributed stochastic variables with a zero mean and a finite *rms*:

$$\overline{\Omega}_\alpha = n\ \langle \omega_\alpha \rangle = n\ (\frac{1}{n}\ \sum_{i=1}^{n} \omega_\alpha^i),$$

$$\sqrt{\overline{\Omega_\alpha^2}} = \sqrt{\overline{\Omega_\alpha\Omega_\alpha} - \overline{\Omega}_\alpha\overline{\Omega}_\alpha\rangle} = \sqrt{n(\langle\sum_{i=1}^{n}\omega_\alpha^{i^2}\rangle - \langle\sum_{i=1}^{n}\omega_\alpha^i\rangle^2)} \tag{9}$$

Without any loss of generality we can estimate the order of $\overline{\Omega_\alpha^2}$ for a particle with the incident direction vector, $\hat{\mathbf{x}}^a = (0,0,1)$. The dot product between the direction vectors, is proportional to the scattering angle, Θ^{MS}:

$$\sqrt{\langle(\hat{\mathbf{x}}^a \cdot \hat{\mathbf{x}}^b)^2\rangle} = cos(\Theta^{MS}) \cong 1 - \frac{\Theta^{MS^2}}{2} \tag{10}$$

Note however, that the rotated direction vector has to be normalized, such that , $\hat{\mathbf{x}}^b \cdot \hat{\mathbf{x}}^b = 1$. We thus have,

$$\hat{\mathbf{x}}^b = \frac{1}{\mathbf{N}}[\mathbf{I} + \overline{\epsilon}]\hat{\mathbf{x}}^a = \frac{1}{\sqrt{1 + \overline{\Omega_1^2} + \overline{\Omega_2^2}}}\,(-\overline{\Omega}_2, \overline{\Omega}_1, 1) \tag{11}$$

with the normalization factor, $\mathbf{N} = \sqrt{1 + \overline{\Omega_1^2} + \overline{\Omega_2^2}}$. The isotropy of the scattering material and the stochastic nature of the process, allows to assume that the effective rotation angles, $\overline{\Omega}_\alpha$, are of the same order. We therefore substitute equation (11) into (10) with the above assumption, $\overline{\Omega_1^2} \cong \overline{\Omega_2^2}$, to obtain an estimate for the rotation angles *rms*:

$$\overline{\Omega_\alpha^2} \cong \frac{\Theta^{MS^2}}{2} \tag{12}$$

Using equation (7) for an arbitrary incident vector, $\hat{\mathbf{x}} = (\hat{x}_1, \hat{x}_2, \hat{x}_3)$, we have,

$$\hat{\mathbf{x}}' = [\mathbf{I} + \overline{\epsilon}]\;\hat{\mathbf{x}} \tag{13}$$

which up to a normalization factor leads to the sought expressions for the direction errors:

$$\delta\hat{\mathbf{x}} = \begin{pmatrix} \overline{\Omega}_3\hat{x}_2 - \overline{\Omega}_2\hat{x}_3 \\ \overline{\Omega}_1\hat{x}_3 - \overline{\Omega}_3\hat{x}_1 \\ \overline{\Omega}_2\hat{x}_1 - \overline{\Omega}_1\hat{x}_2 \end{pmatrix} = \hat{\mathbf{x}} \times \overline{\mathbf{\Omega}} \tag{14}$$

with $\overline{\mathbf{\Omega}} = (\overline{\Omega}_1, \overline{\Omega}_2, \overline{\Omega}_3)$, each with an *rms* of the order of, $\frac{\Theta^{MS}}{\sqrt{2}}$. The sum over stochastic variables is a stochastic variable itself, therefore the average rotation angles, $\overline{\Omega}_\alpha$, fulfill the following relation:

$$\overline{\Omega}_\alpha\overline{\Omega}_\beta = \overline{\Omega_\alpha\Omega_\beta} \tag{15}$$

In view of equation (15), the components of the error matrix are:

$$\langle\delta\hat{x}_\alpha\delta\hat{x}_\alpha\rangle - \langle\delta\hat{x}_\alpha\rangle\langle\delta\hat{x}_\alpha\rangle = \frac{\Theta^{MS^2}}{2}(1 - \hat{x}_\alpha^2) \tag{16}$$

$$\begin{aligned} &\langle\delta\hat{x}_\alpha\delta\hat{x}_\beta\rangle - \langle\delta\hat{x}_\alpha\rangle\langle\delta\hat{x}_\beta\rangle = \\ &\quad\langle(\delta_{\alpha\beta\gamma}\overline{\Omega}_\gamma\hat{x}_\beta - \delta_{\alpha\beta\gamma}\overline{\Omega}_\beta\hat{x}_\gamma)(\delta_{\alpha\beta\gamma}\overline{\Omega}_\alpha\hat{x}_\gamma - \delta_{\alpha\beta\gamma}\overline{\Omega}_\gamma\hat{x}_\alpha)\rangle - \\ &\quad\quad\langle\delta_{\alpha\beta\gamma}\overline{\Omega}_\gamma\hat{x}_\beta - \delta_{\alpha\beta\gamma}\overline{\Omega}_\beta\hat{x}_\gamma\rangle\langle\delta_{\alpha\beta\gamma}\overline{\Omega}_\alpha\hat{x}_\gamma - \delta_{\alpha\beta\gamma}\overline{\Omega}_\gamma\hat{x}_\alpha\rangle \\ &\quad = -\hat{x}_\alpha\hat{x}_\beta\frac{\Theta^{MS^2}}{2} \end{aligned} \tag{17}$$

with, $\delta_{\alpha\beta\gamma}$, the Levi-Civita density tensor.

Let us emphasize again that the error matrix, $V^{ms}_{\alpha\beta}$, is easily transformed to any other set of parameters other than the Cartesian. Using the "propagation error formula" [6],

$$V_{\mu\nu}(\bar{f}) \approx \sum_{\alpha,\beta} \frac{\partial f_\mu}{\partial x_\alpha}\frac{\partial f_\nu}{\partial x_\beta} V_{\alpha\beta}(\bar{x}),$$

one can express the errors on any other parameterization, $\bar{f}$, of the particle trajectory.

An algorithm for the MS error estimate

For a large number of scattering events, $n \gg 1$, the formal solution described above, can be built into an n-independent algorithm for a Monte Carlo estimate of the MS errors.

The problem of a particle traversing any number of radiation lengths, can be described by a random walk (RW), where the errors of a single step are inferred from equation (14), and the different energy loss mechanisms. The 'step length' of the 'walking' particle has to be much smaller than the total distance after n steps. The measured MS angle for a particle traversing 0.1 of a radiation length, was found to agree with Moliere's theory [7]. However, it is common to parameterize the angular error by the Highland formula [2]. Our RW scheme breaks the particle trajectory in the scattering material into n steps, each step approximated by a quasi linear curve of 0.1 of a radiation length. The total trajectory sums up to an n-step walk. With each step is associated a random direction error and thus a position error (displacement). These errors sum up to an end-to-end error on the position and the direction of the emerging particle, compared to a particle traversing the vacuum.

At a given step, i, the errors of the track parameters (the position and the direction of the particle at that step) due to the MS process can be described by a 4 dimensional vector, $\delta\mathbf{v_i} = (\delta x, \delta y, \delta\hat{x}, \delta\hat{y})$, and an average momentum loss, δp. The error after an n step walk is thus described by a sum over the local random vectors $\delta\mathbf{v_i}$:

$$\delta\mathbf{V} = \sum_{i=1}^{n} \delta\mathbf{v_i} \tag{18}$$

where $\delta\mathbf{V}$ is the 4-dimensional error vector that emerges from the n-step walk. The probability density of finding $\delta\mathbf{V}$ between $\delta\mathbf{V}$ and $\delta\mathbf{V} + d^4\delta\mathbf{V}$ can be described by a Gaussian distribution:

$$\mathbf{G}(\delta\mathbf{V})d^4\delta\mathbf{V} = \frac{1}{(2\pi n|\mathbf{M}|)^2}\exp[-\frac{1}{2n}(\delta\mathbf{V} - \langle\delta\mathbf{V}\rangle)^T M^{-1}(\delta\mathbf{V} - \langle\delta\mathbf{V}\rangle)] \tag{19}$$

where M is the end-to-end covariance error matrix given by,

$$M_{\alpha\beta} = \langle\delta\mathbf{v}_\alpha\delta\mathbf{v}_\beta\rangle - \langle\delta\mathbf{v}_\alpha\rangle\langle\delta\mathbf{v}_\beta\rangle \tag{20}$$

with α, β running over the 4 indices of the local vector $\delta\mathbf{v_i}$. The moments , $\langle\delta\mathbf{v}_\alpha\rangle$ and $\langle\delta\mathbf{v}_\alpha\delta\mathbf{v}_\beta\rangle$, are given by the following integrals:

$$\langle\delta\mathbf{v}_\alpha\rangle = \int \delta\mathbf{v}_\alpha \rho(\delta\mathbf{v}) d^4\delta\mathbf{v}$$

$$\langle \delta \mathbf{v}_\alpha \delta \mathbf{v}_\beta \rangle = \int \delta \mathbf{v}_\alpha \delta \mathbf{v}_\beta \rho(\delta \mathbf{v}) d^4 \delta \mathbf{v} \qquad (21)$$

with $\rho(\delta \mathbf{v}) d^4 \delta \mathbf{v}$, the joint probability that the components of a single step vector, $\delta \mathbf{v}_\alpha$, fall in the interval $\delta \mathbf{v}_\alpha + d\delta \mathbf{v}_\alpha$. The integration over these correlated variables is done in the entire 4-dimensional parameter space and due to the correlation of the track parameters, it is non trivial.

The RW approach allows one to calculate these integrals and obtain the end-to-end error matrix, $M_{\alpha\beta}$, based on the knowledge of $\rho(\delta \mathbf{v}) d^4 \delta \mathbf{v}$. The problem is thus reduced to the parameterization of the local errors of each of the entries, $\mathbf{v}_\alpha$, in a single step. The errors in a given step are calculated according to equation (14). The three independent random variables, $\overline{\Omega}_\alpha$, are each of a Gaussian probability density, $\rho(\overline{\Omega}_\alpha) d\overline{\Omega}_\alpha \propto \exp(\frac{-\overline{\Omega}_\alpha^2}{2\sigma(\Theta^{MS})^2})$, with $\sigma(\Theta^{MS})$ parameterized in the Highland fashion [2]:

$$\sigma(\Theta^{MS}) = \frac{0.013}{p} \sqrt{L_\circ} [1. + 0.038 \log(L_\circ)] \qquad (22)$$

The error on the position for each step is determined by equation (14) and the step length. The energy loss at each step, is accounted for by a parameterization based on a fit to the data in [8]. Following this concept, our algorithm is consisted of simulating a RW of a particle with an initial momentum, $p_\circ$, through various numbers of radiation lengths, $L_\circ$. The end-to-end error matrix is estimated by the *rms* of the position and direction errors of the particle as it emerges out of the material.

Results

In this study 1000 'muons' are stepped with a 0.1 radiation length step size, in a RW manner, through various numbers of radiation lengths, all with an initial momentum of 40 GeV and an incident direction of, $\frac{\sqrt{3}}{3}(1,1,1)$.

Our results are shown in figures 1 to 3. In figure 1a-1d we show the accumulated error on the direction and the position of a 40 GeV muon traversing 100 radiation lengths (equivalent to 1.76 m of iron). The distributions are of a Gaussian form with a zero mean and a finite *rms*. In figure 2a-2f we plot the correlations between the track parameters. The long range correlations seen in figure 2a indicate that the particle remembers its direction. However, had we rotated the system of coordinates to the particles momentum axis, i.e. (0,0,1), the correlations would have vanished due to the azimuthal symmetry. In figure 3-a we plot the projected mean scattering angle defined as,

$$\Theta_{rms}^{RW} = cos^{-1}(\sum_{\alpha=1}^{3} \hat{x}_\alpha^{in} \cdot \hat{x}_\alpha^{out}) \qquad (23)$$

with in/out standing for going into/out of the scattering material. In fig. 3-b we plot the average momentum of the emerging muon after its n-step walk. In fig. 3-c we plot the *rms* of the accumulated direction error, and fig. 3-d shows the position errors after traversing the material. Overlaid are functions of the form;

for the mean scattering angle (fig. 3-a),

$$f_a(\Theta^{MS}, L_\circ, p_\circ = 40.) = \frac{\sigma(\Theta^{MS})}{\sqrt{2}} \tag{24}$$

for the momentum after accounting for the energy loss (fig. 3-b),

$$f_b(L_\circ, p_\circ = 40.) = 40. - c_p\ L_\circ \tag{25}$$

for the direction error (fig. 3-c),

$$f_c(\Theta^{MS}, L_\circ, p_\circ = 40.) = c(\rho_{\hat{x}\hat{y}})\ \sqrt{1-\hat{x}^2}\ \frac{\sigma(\Theta^{MS})}{\sqrt{2}} \tag{26}$$

and for the position error (fig. 3-d),

$$f_d(\Theta^{MS}, L_\circ, p_\circ = 40.) = c(\rho_{\hat{x}\hat{y}})\ \sqrt{1-\hat{x}^2}\ \frac{\sigma(\Theta^{MS})}{\sqrt{2}}\ L_\circ \tag{27}$$

all with $\sigma(\Theta^{MS})$ of equation (22). The parameter , c_p, in equation (25) is ≈ 0.0155, which implies ≈ 0.6 GeV energy loss per radiation length for a 40 GeV muon traveling in iron. The parameter $c(\rho_{\hat{x}\hat{y}}) \approx 0.8$ in equations (26) and (27), is related to the correlation coefficient of the linear dependence between the errors in each direction.

To summarize, based on the stochastic nature of the MS process, we have formally derived a closed form of the errors inflicted on the track parameters for a particle traversing the detector material. This error parameterization is used to simulate a random walk type of solution to estimate the end-to-end error matrix. The errors are well described by the Highland formula.

Acknowledgements

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Figure captions

Fig. 1 Distribution of the errors of a 40 GeV muon traversing 100 radiation lengths. Figure 1-a and 1-b show the direction errors, and 1-c and 1-d the position errors.

Fig. 2 Correlations between errors of a 40 GeV muon traversing 100 radiation lengths. Figure 2a shows the correlations, $\langle \delta\hat{x}\delta\hat{y} \rangle$, fig. 2-b shows the correlations , $\langle \delta x\delta y \rangle$, 2-c and 2-d depict respectively the correlations, $\langle \delta x\delta\hat{x} \rangle$ and $\langle \delta y\delta\hat{y} \rangle$. In figures 2-e and 2-f we plot the correlations, $\langle \delta x\delta\hat{y} \rangle$ and $\langle \delta y\delta\hat{x} \rangle$ respectively.

Fig. 3 In fig. 3-a we show the projected mean scattering angle, Θ_{rms}^{RW}, of equation (23), as a function of, $L_\circ$, in fig. 3-b we plot the momentum of the muon after traversing the material with $L_\circ$, radiation lengths. In fig. 3-c and fig. 3-d we show the direction error and the position errors of the emerging muon after the n-step walk. Overlaid are the functions of equations (24) - (27).

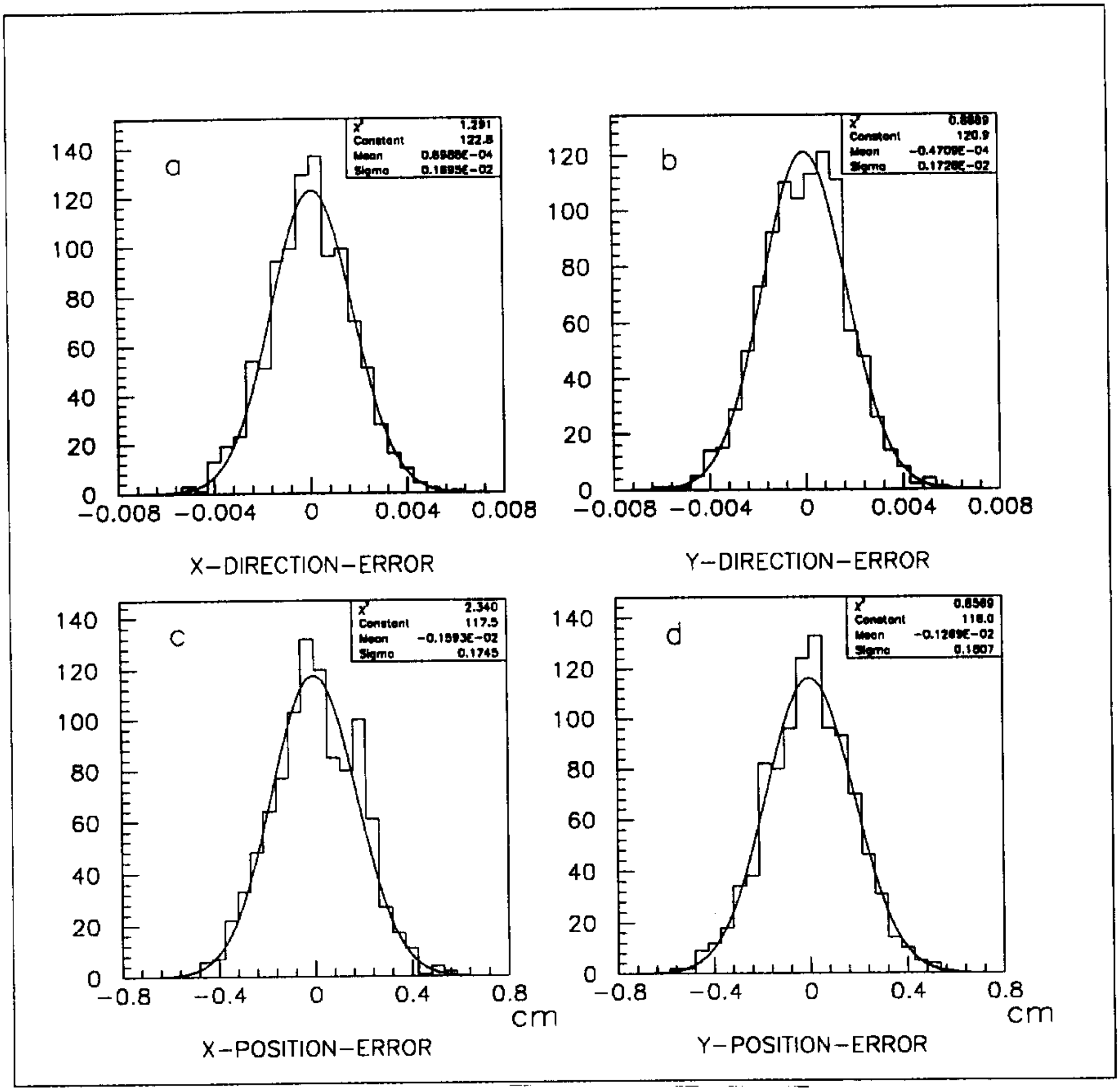
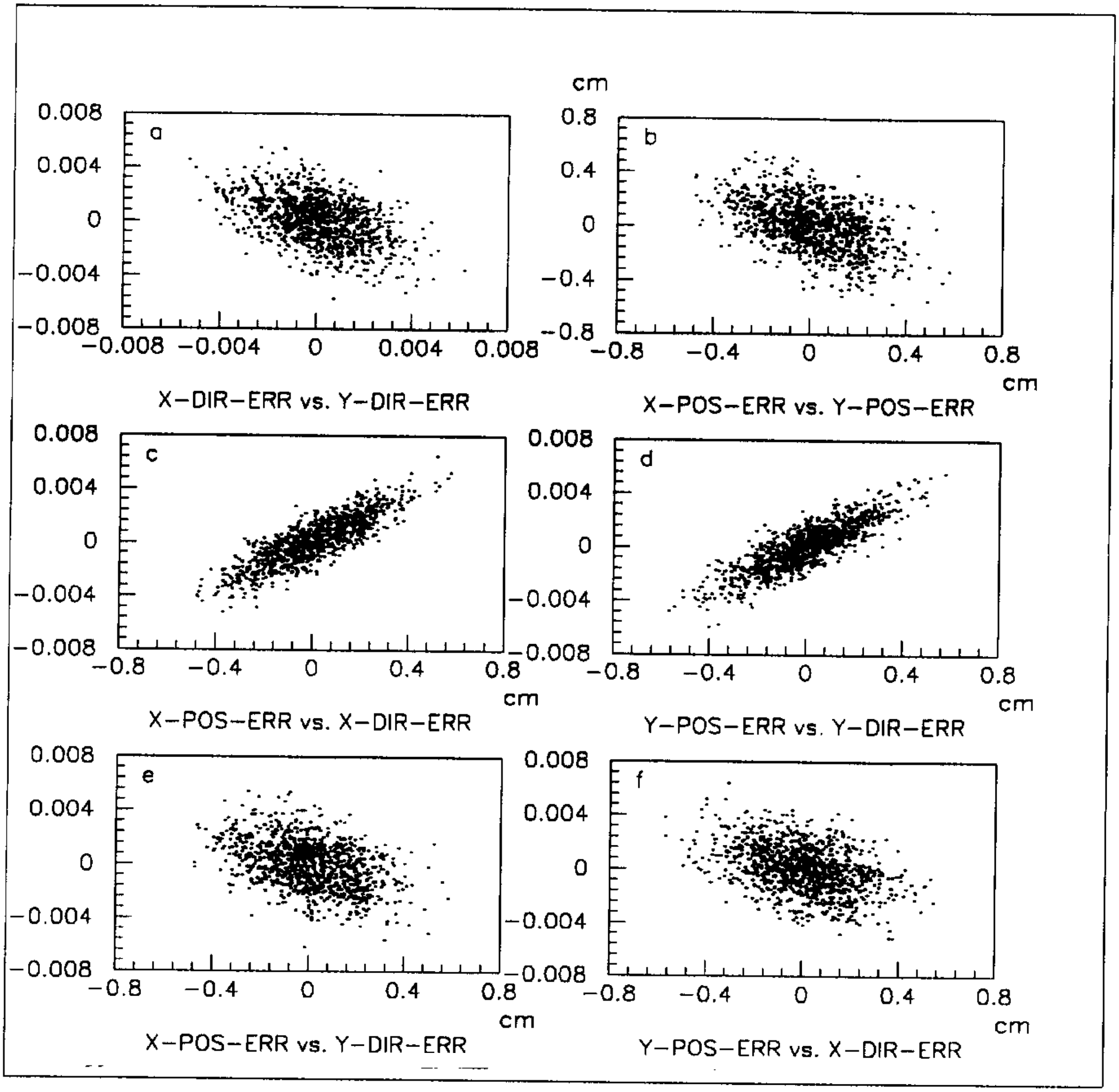
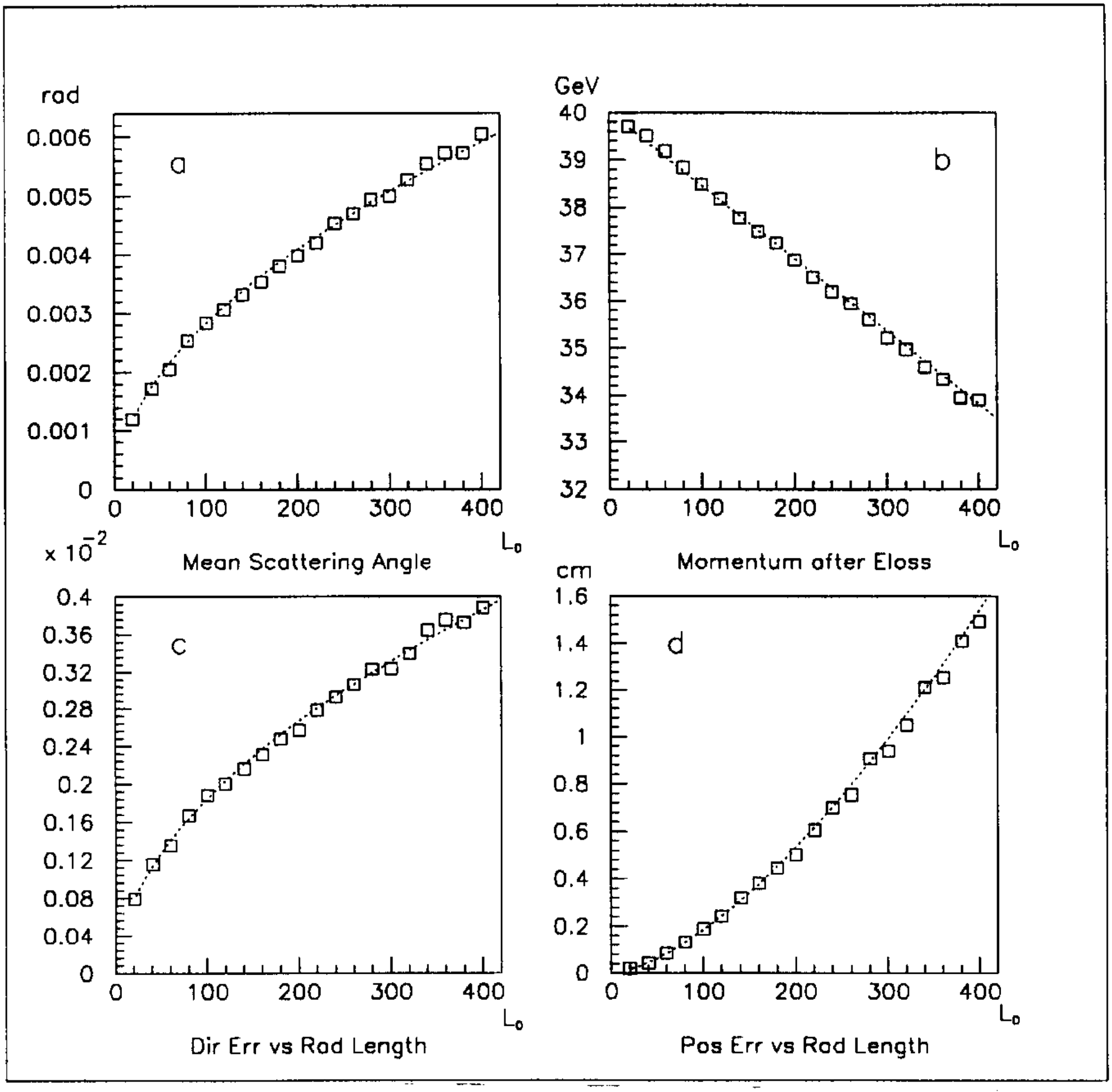

a
b
c
d
X-DIRECTION-ERROR
Y-DIRECTION-ERROR
X-POSITION-ERROR
Y-POSITION-ERROR
cm
cm

Fig. 1



a
b
c
d
e
f
cm
0.008
0.004
0
−0.004
−0.008
−0.008
−0.004
0.004
0.008
0.8
0.4
−0.4
−0.8
X−DIR−ERR vs. Y−DIR−ERR
X−POS−ERR vs. Y−POS−ERR
X−POS−ERR vs. X−DIR−ERR
Y−POS−ERR vs. Y−DIR−ERR
X−POS−ERR vs. Y−DIR−ERR
Y−POS−ERR vs. X−DIR−ERR

Fig. 2



rad
a
Mean Scattering Angle
GeV
b
Momentum after Eloss
$\times 10^{-2}$
c
Dir Err vs Rad Length
cm
d
Pos Err vs Rad Length
L_0

Fig. 3